Exercise 11

Determine which of the following matrices are invertible:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 3 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 & 3 \\ -1 & 1 & 19 \\ 2 & 3 & \pi \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Solution

Calculate the determinants of A, B, and C.

$$\begin{aligned} \det A &= \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 3 & 3 \end{vmatrix} \\ &= 1\begin{vmatrix} 1 & 1 \\ 3 & 3 \end{vmatrix} - 0\begin{vmatrix} 2 & 3 \\ 3 & 3 \end{vmatrix} + 0\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} \\ &= 1[(1)(3) - (1)(3)] - 0[(2)(3) - (3)(3)] + 0[(2)(1) - (3)(1)] \\ &= 1(0) - 0(-3) + 0(-1) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \det B &= \begin{vmatrix} 0 & 0 & 3 \\ -1 & 1 & 19 \\ 2 & 3 & \pi \end{vmatrix} \\ &= 0\begin{vmatrix} 1 & 19 \\ 3 & \pi \end{vmatrix} - 0\begin{vmatrix} -1 & 19 \\ 2 & \pi \end{vmatrix} + 3\begin{vmatrix} -1 & 1 \\ 2 & 3 \end{vmatrix} \\ &= 0[(1)(\pi) - (19)(3)] - 0[(-1)(\pi) - (19)(2)] + 3[(-1)(3) - (1)(2)] \\ &= 0(\pi - 57) - 0(-\pi - 38) + 3(-5) \\ &= -15 \end{aligned}$$

$$\begin{aligned} \det C &= \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \\ &= (1)(1) - (1)(1) \\ &= 0 \end{aligned}$$

Since only $\det B \neq 0$, only B is invertible.